Pearson Edexcel

Mark Scheme (Results)

Summer 2022

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers normally score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = ...$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = ...$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x = ...$

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$ leading to $x = ...$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1	**Method 1** $16 = a + (5-1)d \quad (= a + 4d)$ or $301 = a + (100-1)d \quad (= a + 99d)$	M1
	$95d = 285$	M1
	$d = 3 \qquad a = 4$	A1
	Method 2 $301 - 16 = 295 \Rightarrow 295 = (100-5)d$	[M1
	$\Rightarrow d = \frac{285}{95} = (3)$	M1
	$d = 3 \quad a = 4$	A1]
	<u>**Sum to 50 terms**</u> <u>Uses</u> $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ $S_{50} = \frac{50}{2}[2\times"4"+"3"(50-1)] = 3875$	M1A1
	ALT <u>Uses</u> $S_n = \frac{n}{2}(a + l)$	[5]
	50th term = "4" + 49 ×"3" (=151) **and** $S_{50} = \frac{50}{2}('4'+'151') = 3875$	[M1A1]
		Total 5 marks

Mark	Additional Guidance
Method 1	
M1	At least, one fully correct equation.
M1	Solves their linear equations simultaneously, allow one arithmetical or processing slip to eliminate *a* (or possibly *d*).
A1	Both *a* and *d* correct.
Method 2	
M1	Uses the difference of the two terms $(301-16)$ together with the difference in position $(100-5)$ and equates to form the equation $285=95d$ This may be implied by their working.
M1	Solves the equation, allow one arithmetical or processing slip to find a value for *d*
A1	Both *a* and *d* correct.
NOTE: If you see the correct values of *a* and *d* that come from a method without errors, award **M1M1A1**	
Summation	
M1	Substitution of their values for *a* and *d* into the correct summation formula. **OR** They must use the formula for the 50th term correctly using their values and use the correct formula for first plus last with their values.
A1	For 3875

Question number	Scheme	Marks
2(a)	$(a=)2t-3$	M1 A1 [2]
(b)	$(s=\int(t^2-3t+4)\,\mathrm{d}t)$ $(s=)\dfrac{t^3}{3}-\dfrac{3}{2}t^2+4t(+C)$ oe $(t=2,\ s=7)$ $\dfrac{2^3}{3}-\dfrac{3}{2}\times2^2+4\times2+C=7\Rightarrow C=\dfrac{7}{3}$ $\dfrac{4^3}{3}-\dfrac{3}{2}\times4^2+4\times4+'\dfrac{7}{3}'=\dfrac{47}{3}$ oe **ALT** Displacement $-7=\int_2^4\left(x^2-3x+4\right)\,\mathrm{d}x$ $=\left[\dfrac{x^3}{3}-\dfrac{3x^2}{2}+4t\right]_2^4$ $=\left[\dfrac{4^3}{3}-\dfrac{3\times4^2}{2}+4\times4\right]-\left[\dfrac{2^3}{3}-\dfrac{3\times2^2}{2}+4\times2\right]$ $=\dfrac{47}{3}$	 M1 A1 dM1 M1A1 [5] [M1A1 dM1M1 A1]
		Total 7 marks

Part	Mark	Additional Guidance
2(a)	M1	For a minimally acceptable attempt at differentiation (see general guidance), no power of t must increase.
	A1	Correct expression with or without $a = \ldots$
(b)	M1	For a minimally acceptable attempt at integration (see general guidance), no power of t must decrease. + C is not necessary for this mark.
	A1	Fully correct integration. C is not necessary for this mark.
	dM1	Correct substitution of $t = 2$ into **their** integrated expression, correctly equated to 7 and an attempt to rearrange to find C. **Note: This mark is dependent on the first M mark in (b)**
	M1	Substitution of $t = 4$ correctly into their expression for s, provided it is a changed expression from v
	A1	Correct value oe must be an exact value.
	ALT	
	M1	For a minimally acceptable attempt at integration (see general guidance), no power of t must decrease.
	A1	Fully correct integration
	dM1	For the correct limits between $t = 4$ and $t = 2$ **and** equates to $d - 7$ or equivalent
	M1	Substitutes the limits to evaluate the integral
	A1	Correct value oe must be an exact value.

Question number	Scheme	Marks
3 (a)	$(\frac{1}{2} \times 4^2 \times \theta = 2\pi)$ $\frac{1}{2} \times "16" \times \theta = 2\pi \Rightarrow \theta = ...$ $\theta = \frac{2\pi}{8} = \frac{\pi}{4}$ oe	 M1 A1 [2]
(b)	(arc SR =) $4 \times " \frac{\pi}{4} " (= \pi)$ $(4 + 4 + \pi) = 11.1$ (cm) Accept awrt 11.1 (cm)	M1 A1ft [2]
(c)	$\frac{1}{2} \times (1) \times 4 \times \sin " \frac{\pi}{4} " \quad (= \sqrt{2})$ $2\pi - \sqrt{2}$ oe	M1 A1 [2]
		Total 6 marks

Part	Mark	Additional Guidance
(a)	M1	Use of the correct formula for the area of sector, with correct substitution of 4 and 2π, an attempt to square r and to rearrange to find a value for θ. The rearrangement has to be correct for this mark.
	A1	Accept any valid exact equivalent. Do not accept answers in degrees or answers not given exactly for A1. However, 45° or 0.785(3981…) stated with no method - award implied M1.
(b)	M1	Use of correct formula for length of an arc with the correct radius and "their θ" (allow working in degrees for this mark) from part (a). Do **not** allow $\theta = \frac{\pi}{2}$ or 90°
	A1ft	Allow **ft** of their angle from part (a), if complete method shown and if their answer is correct to 3sf. **Note this is ft mark. Check working carefully.**
(c)	M1	Use of correct formula for area of a triangle with correct sides substituted and "their θ" (allow working in degrees) from part (a). Do **not** allow $\theta = \frac{\pi}{2}$ or 90°
	A1	For the correct answer shown.

Question number	Scheme	Marks
4 (a) (i)	$4x + 8$ oe e.g. accept $x+(x+4)+x+(x+4)$	B1
(ii)	$x\ (x + 4)$ *or* $x^2 + 4x$	B1 [2]
(b)	$4x + 8 < 30$ ***or*** $x^2 + 4x > 12$	M1
	$x < 5.5$ oe	B1
	$(x^2 + 4x - 12 > 0\ )$ $(x + 6)(x - 2)(> 0)$	M1
	$x = 2$ $[x = -6]$	A1
	$2 < x < 5.5$	B1ft [5]

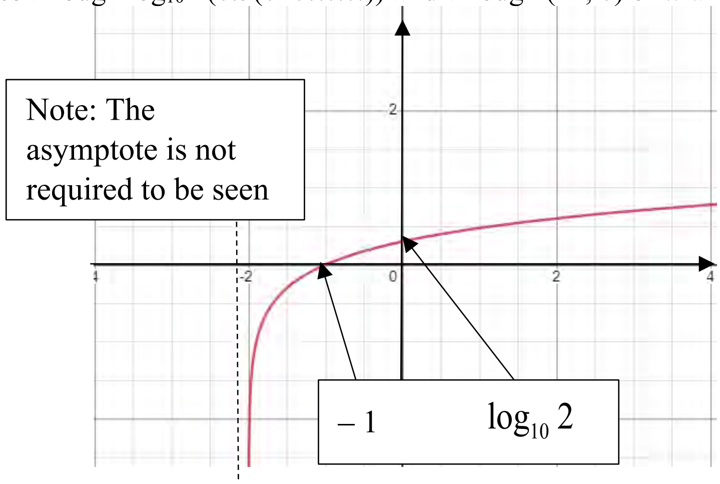
Part	Mark	Additional Guidance
(b)	**Note: Do not** accept $\leqslant$ or $\geqslant$ in place of $<$ and $>$	
	M1	**Either** their linear expression < 30 (seen or implied in later work) ***or*** their quadratic expression > 12 (seen or implied in later work).
	B1	$x < 5.5$ oe allow implied M1 if $(4x + 8)$ not seen in earlier work. Allow $x = 5.5$ for this mark.
	M1	A minimally acceptable (see general guidance) attempt to factorise, use the formula or complete the square to solve their 3TQ.
	A1	For the correct critical value $x = 2$. Ignore $x = -6$ for this mark. This may be given as an inequality. Award the mark for 2 seen even if it is embedded in a spurious inequality.
	B1ft	ft their values for x, provided the inequality is written correctly and is correct for their values. Must be in the form $a < x < b$ Both values of x must be positive. Their value of '2' must be less than their value from the linear solution. i.e < their 5.5

Question number	Scheme	Marks
5 (a)	$4e^{4x}(6x+2)^{\frac{3}{2}}+\frac{3}{2}.6.e^{4x}(6x+2)^{\frac{1}{2}}=\left[4e^{4x}(6x+2)^{\frac{3}{2}}+9e^{4x}(6x+2)^{\frac{1}{2}}\right]$	M1 A1 A1
	$e^{4x}\sqrt{6x+2}(24x+17)$	dM1 A1 [5]
(b)	$\frac{3\cos 3x(2x-4)^3-\sin 3x.2.3.(2x-4)^2}{\left[(2x-4)^3\right]^2}=\left[\frac{3\cos 3x(2x-4)^3-6\sin 3x(2x-4)^2}{(2x-4)^6}\right]$	M1 A1 A1
	OR $\frac{3\cos 3x(2x-4)^3-\left(24x^2-96x+96\right)\sin 3x}{(2x-4)^6}$	
	ALT using product rule $\sin 3x(2x-4)^{-3}$	
	$3\cos 3x(2x-4)^{-3}+\sin 3x(-3)(2)(2x-4)^{-4}=\left[3\cos 3x(2x-4)^{-3}-6\sin 3x(2x-4)^{-4}\right]$	M1 A1 A1 [3]
	Total	**8 marks**

Part	Mark	Additional Guidance
(a)	M1	Use of product rule to give an expression of the form $pe^{4x}(6x+2)^{\frac{3}{2}}+qe^{4x}(6x+2)^{\frac{1}{2}}$ where $p = 1$ or 4, $q > 1$. There must be a + sign between terms.
	A1	Either term correct. Note that simplification is not required for this mark.
	A1	Both terms correct. Note that simplification is not required for this mark.
	dM1	Obtains an expression of the form $e^{4x}\left(\sqrt{6x+2}\right)(Ax+B)$ where $A, B \neq 0$ You **must** check their working here that it follows from their attempt at product rule. There must be an intermediate step seen as follows for example: $4e^{4x}(6x+2)^{\frac{3}{2}}+9e^{4x}(6x+2)^{\frac{1}{2}}=e^{4x}(6x+2)^{\frac{1}{2}}\left(4\left(6x+2\right)+9\right)$ **Note: This mark is dependent on the first M Mark**
	A1	Correct $A = 24$ and correct $B = 17$
(b)	M1	Attempt at the quotient rule. Numerator must be the **difference** of two terms (either way round) of the form $k\cos 3x(2x-4)^3 - l\sin 3x(2x-4)^2$, $k = \pm 3$ $l \geqslant 3$ OR accept: (terms wrong way around) $l\sin 3x(2x-4)^2 - k\cos 3x(2x-4)^3$, $k = \pm 3$ $l \geqslant 3$ Denominator must be of the form $(2x-4)^6$ or accept $\left[(2x-4)^3\right]^2$
	A1	Either term correct, either way round.
	A1	Fully correct
	ALT	
	M1	Use of product rule to give an expression of the form $k\cos 3x(2x-4)^{-3} - l\sin 3x(2x-4)^{-4}$, $k = \pm 3$ $l \geqslant 3$ Accept for this mark $l\sin 3x(2x-4)^{-4} - k\cos 3x(2x-4)^{-3}$
	A1	Either term correct.
	A1	Fully correct
	NB: Ignore any subsequent simplification of their derivative.	

Question number	Scheme		Marks
6 (a)	**Method 1** $\left(\dfrac{a+\sqrt{5}}{\sqrt{5}-2}\times\dfrac{\sqrt{5}+2}{\sqrt{5}+2}\right)=\dfrac{\sqrt{5}a+2a+5+2\sqrt{5}}{(1)}$ $=\left[2a+5+(2+a)\sqrt{5}\right]$ $a = 3$	**Method 2** $((11+5\sqrt{5})(\sqrt{5}-2))$ $11\sqrt{5}-22+25-10\sqrt{5}$ $a = 3$	M1 A1 [2]
(b)	(angle PQR =) 45° $\dfrac{x+3}{\sin 45°}=\dfrac{x}{\sin 30°}\Rightarrow\dfrac{x+3}{\frac{\sqrt{2}}{2}}=\dfrac{x}{\frac{1}{2}}\Rightarrow x+3=\sqrt{2}x$ $\Rightarrow 3=x\left(\sqrt{2}-1\right)\Rightarrow x=\dfrac{3}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}\Rightarrow x=...$ $x=3+3\sqrt{2}$ **OR** $\dfrac{x+3}{\sin 45°}=\dfrac{x}{\sin 30°}\Rightarrow\dfrac{x+3}{\frac{\sqrt{2}}{2}}=\dfrac{x}{\frac{1}{2}}\Rightarrow \sqrt{2}x+3\sqrt{2}=2x$ $x\left(2-\sqrt{2}\right)=3\sqrt{2}\Rightarrow x=\dfrac{3\sqrt{2}}{2-\sqrt{2}}\times\dfrac{2+\sqrt{2}}{2+\sqrt{2}}\Rightarrow x=$ $x=3+3\sqrt{2}$		M1 M1 A1* cso [3] [M1 M1 A1]
(c)	$\frac{1}{2}\times(3+3\sqrt{2})\times(6+3\sqrt{2})\times\sin(180-30-45)°$ $\dfrac{1}{2}\times(3+3\sqrt{2})\times(6+3\sqrt{2})\times\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)$ $\dfrac{9}{8}\left(4\sqrt{6}+4\sqrt{2}+3\sqrt{12}+6\right)$ or $\dfrac{1}{8}\left(36\sqrt{6}+36\sqrt{2}+54\sqrt{12}+54\right)$ oe with two out of four terms correct. $\dfrac{9}{8}\left(4\sqrt{6}+4\sqrt{2}+6\sqrt{3}+6\right)$		M1 dM1 A1 cao [3]
			Total 8 marks

Part	Mark	Additional Guidance
Throughout this question allow missing degree signs.		
(a)	M1	Any three out of four terms correct in the numerator of method 1 or in the expansion of method 2. If they use method 1 we must see the expansion come from multiplying numerator and denominator by $\sqrt{5}+2$
	A1	Cao If the answer $a = 3$ is stated without sufficient working seen award M0A0.
(b)	M1	For using a correct sine rule, either way around, using the exact values of sin 45 and sin 30, and attempting to simplify to $kx+3=\sqrt{2}x$ where k is a constant. Allow $\frac{kx}{2}+\frac{3}{2}=\frac{\sqrt{2}x}{2}$ for this mark
	M1	For factorising their $x+3=\sqrt{2}x$ [which must have come from using sine rule] to obtain $3=x\left(\sqrt{2}-k\right)$, rearranging and multiplying numerator and denominator by $\sqrt{2}+k$ This is a given answer – this step must be seen explicitly
	A1*	cso, no errors or emissions.
(c)	M1	Follow general guidance, if area formula is quoted, allow one slip in substitution. If formula not quoted, must be a fully correct substitution. For this mark only allow the angle to be $(180-30-45)^{\circ}$ If they work in terms of x and obtain $\frac{1}{2}\times x\times(x+3)\times\sin 105^{\circ}$ then do not award the first M mark until they have substituted the given value of $3+3\sqrt{2}$ for x.
	dM1	Replaces $\sin 105^{\circ}$ with the given value and attempts to multiply out. The mark can be awarded for any two terms correct, four terms must be present.
	A1cao	Must be in the given form with the terms in any order.

Question number	Scheme		Marks
7 (a)	Shape of the curve, **crossing** negative x-axis and positive y-axis with the asymptotic nature of the curve shown.		B1
	passes through $\log_{10}2$ (0.3(010……)) **and** through (–1, 0) on x-axis		B1 [2]
			
(b)	$\left(2\log_a 4 + 2\log_a 4^2\right)$ $2\log_a 4 + 4\log_a 4$ or $(2\log_a 2^2 + 2\log_a 2^4)$ $4\log_a 2 + 8\log_a 2$		M1
	$\log_a 4 = \frac{1}{6}$ or $\log_a 2 = \frac{1}{12}$ oe		dM1
	$a^{\frac{1}{6}} = 4$ or $a^{\frac{1}{12}} = 2$		
	$a = 4096$		A1
	ALT		
	$(2)\log_a 64 = 1 \Rightarrow \log_a 64 = \frac{1}{2}$		[M1
	$\Rightarrow a^{\frac{1}{2}} = 64$		dM1
	$\Rightarrow a = 4096$		A1] [3]
(c)	$\log_q 16 \rightarrow \frac{\log_2 16}{\log_2 q}$	$\log_2 q \rightarrow \frac{\log_q q}{\log_q 2}$ or $\frac{1}{\log_q 2}$	M1
	$\left(5\frac{\log_2 16}{\log_2 q} + 4\log_2 q = 24\right)$ $20 + 4(\log_2 q)^2 = 24\log_2 q$	$\left(5\log_q 16 + 4\frac{\log_q q}{\log_q 2} = 24\right)$ $20(\log_q 2)^2 + 4 = 24\log_q 2$	M1 A1
	$(\log_2 q - 5)(\log_2 q - 1)(=0)$	$(5\log_q 2 - 1)(\log_q 2 - 1)(=0)$	dM1
	$(\log_2 q = 5) \rightarrow q = 32$ $(\log_2 q = 1) \rightarrow q = 2$	$\left(\log_q 2 = \frac{1}{5}\right) \rightarrow q = 32$ $(\log_q 2 = 1) \rightarrow q = 2$	M1 A1 [6]
			Total 11 marks

Part	Mark	Additional Guidance
(a)	B1	The curve must not bend back on itself and should cross both axes once on negative x and once on positive y. It needs to demonstrate an asymptotic nature. Accept an asymptote drawn correctly as well as their curve.
	B1	It is enough for –1 to be indicated on the x-axis with the curve passing through this point and for $\log_{10}2$ to be marked on the y- axis (decimal 0.3…… allowed) There must be a curve drawn (even incorrectly) to achieve this mark. **Do not** accept intersections unless they are marked on the graph in the correct place.
(b)	M1	For the use of the power rule with logs to obtain $2\log_a 4$ from $\log_a 16$ OR $4\log_a 2$ from $\log_a 16$
	dM1	A **correct** rearrangement of their equation to obtain for example $a^{\frac{1}{6}} = 4$ or $a^{\frac{1}{12}} = 2$ or even of the type $a^1 = 64^2$ There are many ways of completing this. Look for a correct value for the corresponding correct power of a. **This mark is dependent on the first M mark**. Look for correct log work for their values.
	A1	For $a = 4096$
ALT	M1	For use of the correct log rule to obtain $\log_a 64$
	dM1	A correct rearrangement of their equation to obtain $a^{\frac{1}{2}} = 64$ **This mark is dependent on the first M mark**. Look for correct log work for their values.
	A1	For $a = 4096$
(c)	M1	For a fully correct change of base of log to either base 2 or base q
	M1	For an attempt to rearrange and form a 3TQ quadratic. Allow one arithmetical slip only.
	A1	For the correct 3TQ.
	dM1	**Dependent on the second M mark** – an acceptable attempt to solve the quadratic equation – see general guidance.
	M1	For a correct application using either of their solutions moving from log form to exponential form. **Note this is an independent mark.**
	A1	**Both** correct solutions.

Question number	Scheme	Marks
8(a)	$x^3+3x^2y+3xy^2+y^3$	B1 [1]
(b)	$\alpha+\beta=-\frac{3}{2}$ **and** $\alpha\beta=\frac{4}{2}$ oe	B1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1
	$\left("-\frac{3}{2}"\right)^3-3("2")\left("-\frac{3}{2}"\right)$ oe	dM1
	$\frac{45}{8}$ oe	A1 [4]
(c)	$\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=$	
	$\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}=\frac{\left("\frac{45}{8}"\right)}{"2"^2}$	M1
	$\frac{45}{32}$ oe	A1ft
	$\frac{\alpha}{\beta^2}\times\frac{\beta}{\alpha^2}=\frac{1}{\alpha\beta}=\frac{1}{'2'}$	
	$\frac{1}{2}$ oe	B1ft
	$x^2-"\frac{45}{32}"x+"\frac{1}{2}"(=0)$	M1
	$32x^2-45x+16=0$ oe	A1 [5]
		Total 10 marks

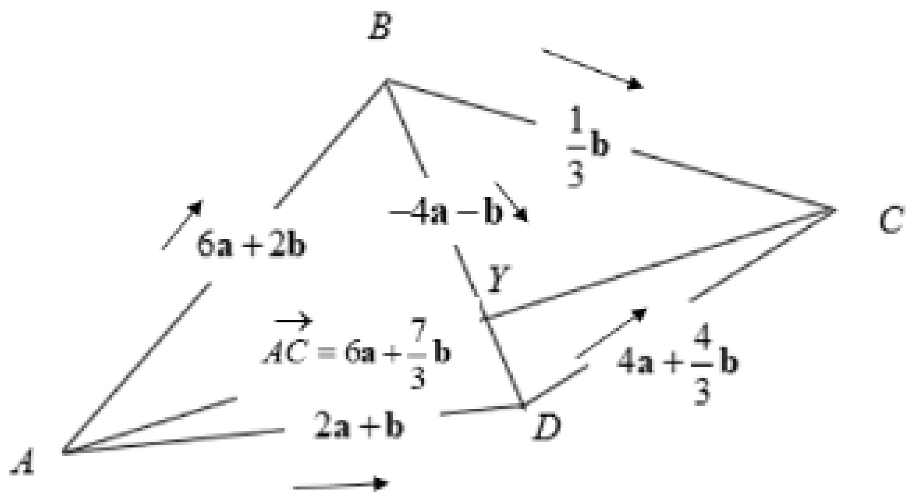
Part	Mark	Additional Guidance
(a)	B1	For the correct expansion simplified or un-simplified. For example, allow $x^3+2x^2y+x^2y+xy^2+2xy^2+y^3$
(b)	B1	Product and sum both correct, written explicitly or used later in working.
	M1	An attempt to rearrange their expression from part (a) to achieve as a minimum $\alpha^3+\beta^3=(\alpha+\beta)^3\pm3\alpha\beta(\alpha+\beta)$ **Note:** Accept alternative algebraic arrangements of $\alpha^3+\beta^3$ but please check carefully that the algebra is correct.
	dM1	Substitution of **their** sum and product into an expression which must be of the form $(\alpha+\beta)^3\pm3\alpha\beta(\alpha+\beta)$ **Note:** If they substitute correctly into alternative arrangements check that they are correct.
	A1	For $\frac{45}{8}$ oe
(c)	M1	Correct algebra to obtain $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$ **and** substitution of **their** values for the sum and product.
	A1ft	For $\frac{45}{32}$ oe **Note:** only follow through their values for product and sum into **correct algebra.**
	B1ft	For the correct algebra and substitution of their $\alpha\beta=2$ to obtain $\frac{\alpha}{\beta^2}\times\frac{\beta}{\alpha^2}=\frac{1}{\alpha\beta}=\frac{1}{'2'}$ ft their $\alpha\beta$
	M1	Use of their sum and product to correctly form an expression as shown, $x^2-"\frac{45}{32}"x+"\frac{1}{2}"(=0)$ Allow missing = 0 for this mark or even = y for this mark.
	A1	Correct equation or any multiple of it, for example , $64x^2-90x+32=0$ The coefficients, must be integers.
	ALT Starts with $\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=0\Rightarrow x^2-x\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right)+\frac{\beta}{\alpha^2}\times\frac{\alpha}{\beta^2}=0$	
	M1	Correct algebra to obtain $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$ **and** substitution of **their** values for the sum and product.
	A1ft	For $\frac{45}{32}$ oe **Note:** only follow through their values for product and sum into **correct algebra.**
	B1ft	For the correct algebra and substitution of their $\alpha\beta=2$ to obtain $\frac{\alpha}{\beta^2}\times\frac{\beta}{\alpha^2}=\frac{1}{\alpha\beta}=\frac{1}{'2'}$ ft their $\alpha\beta$
	M1	For $x^2-x\left('\frac{45}{32}'\right)+'\frac{1}{2}'=0$ Accept this expression on it's own or even = y for this mark
	A1	Correct equation or any multiple of it, for example , $64x^2-90x+32=0$ The coefficients, must be integers.

Question number	Scheme	Marks
9 (a)	$\overrightarrow{AB}=\overrightarrow{AD}+\overrightarrow{DB}=2\mathbf{a}+\mathbf{b}-\ -4\mathbf{a}-\mathbf{b}\ =6\mathbf{a}+2\mathbf{b}$ or $2\ 3\mathbf{a}+\mathbf{b}$ oe	B1
	$\overrightarrow{DC}=\overrightarrow{DB}+\overrightarrow{BC}=4\mathbf{a}+\mathbf{b}+\frac{1}{3}\mathbf{b}=\left(4\mathbf{a}+\frac{4}{3}\mathbf{b}\right)$ or $\frac{4}{3}(3\mathbf{a}+\mathbf{b})$	B1
	$\overrightarrow{AB}=\frac{3}{2}\overrightarrow{DC}$ so $\overrightarrow{AB}$ is a multiple of $\overrightarrow{DC}$	M1
	Conclusion: Therefore, *DC* parallel to *AB*)	A1 [4]
(b)	$\overrightarrow{AC}=2\mathbf{a}+\mathbf{b}+'\left(4\mathbf{a}+\frac{4}{3}\mathbf{b}\right)'=\left[6\mathbf{a}+\frac{7}{3}\mathbf{b}\right]$	M1
	$\overrightarrow{AY}=\lambda“\overrightarrow{AC}”$ $(=\lambda”(6\mathbf{a}+\frac{7}{3}\mathbf{b})”)$	M1
	$\overrightarrow{AY}=\overrightarrow{AD}+\mu\overrightarrow{DB}$ $(=2\mathbf{a}+\mathbf{b}+\mu(4\mathbf{a}+\mathbf{b}))$ or $\overrightarrow{AY}=“\overrightarrow{AB}”+\alpha\overrightarrow{BD}$ $(=“6\mathbf{a}+2\mathbf{b}”+\alpha(-4\mathbf{a}-\mathbf{b}))$	M1
	$2+4\mu=6\lambda$, $1+\mu=\frac{7}{3}\lambda$ or $6-4\alpha=6\lambda$, $1+\mu=\frac{7}{3}\lambda$	M1
	$\lambda=\frac{3}{5}$ or $\mu=\frac{2}{5}$ or $\alpha=\frac{3}{5}$	A1
	$\overrightarrow{AY}=\frac{18}{5}\mathbf{a}+\frac{7}{5}\mathbf{b}$ or $\overrightarrow{AY}=\frac{1}{5}(8\mathbf{a}+7\mathbf{b})$	A1cao [6]
		Total 10 marks

Part	Mark	Additional Guidance
(a)	B1	For stating a valid vector for $\overrightarrow{AB}=6\mathbf{a}+2\mathbf{b}$ [accept simplified or unsimplified] **NB: This is an M mark in Epen**
	B1	For stating a valid vector for $\overrightarrow{DC}=\frac{4}{3}(3\mathbf{a}+\mathbf{b})$ [accept simplified or unsimplified] **NB: This is an A mark in Epen**
	M1	For comparing their two vectors and establishing that $\overrightarrow{DC}$ and $\overrightarrow{AB}$ are multiples of each other .i.e. $\overrightarrow{AB}=\frac{3}{2}\overrightarrow{DC}$ oe **NB** This statement must be correct! **<u>Incorrect directions.</u>** Award this mark if they are comparing for example; $\overrightarrow{CD}$ and $\overrightarrow{AB}$ or $\overrightarrow{CD}$ and $\overrightarrow{BA}$ and ft their incorrect direction for the M mark.

	A1ft	For a conclusion that the vectors are therefore parallel. Allow for example $\overrightarrow{AB} = -6\mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{DC} = \frac{4}{3}(3\mathbf{a} + \mathbf{b})$ are therefore parallel Accept a very brief conclusion even if it is just the word 'shown' or 'QED' or even '#' or a tick.
(b)	Part (b) can be done in a number of different ways – use the following as general principles to mark, referring also to the scheme and additional guidance for the example given, to determine when each mark should be awarded	
	M1	For any simplified or un-simplified vector along AC $\overrightarrow{AC} = 2\mathbf{a} + \mathbf{b} + '\left(4\mathbf{a} + \frac{4}{3}\mathbf{b}\right)' = \left[6\mathbf{a} + \frac{7}{3}\mathbf{b}\right]$ For this mark allow $\overrightarrow{CA} = -6\mathbf{a} - \frac{7}{3}\mathbf{b}$ as it is possible to use this to find $\overrightarrow{AY}$ This mark can be implied by a correct vector for $\overrightarrow{AY}$ e.g. $\overrightarrow{AY} = \lambda\left[6\mathbf{a} + \frac{7}{3}\mathbf{b}\right]$
	M1	Uses their vector for $\overrightarrow{AC}$ to write a vector for $\overrightarrow{AY}$ by introducing a parameter e.g. $\overrightarrow{AY} = \lambda\left[6\mathbf{a} + \frac{7}{3}\mathbf{b}\right]$
	M1	States a second valid vector path for $\overrightarrow{AY}$ introducing a second parameter that can be used with the first vector path [to solve an equation to find their parameters]. Note: this mark is awarded only for stating the vector path, but it must be valid – i.e. able to be used to find the parameters. This second parameter cannot be 1 – their first parameter.
	dddM1	Equating components and reaching a value for any of their parameters. Allow arithmetical slips in processing their simultaneous equations. This is dependent on all previous method marks.
	A1	Any one correct value for one of their two parameters.
	A1	For $\overrightarrow{AY} = \frac{18}{5}\mathbf{a} + \frac{7}{5}\mathbf{b}$

USEFUL SKETCH



Question number	Scheme	Marks
10 (a)	$\sin(A+B)+\sin(A-B)=\sin A\cos B+\sin B\cos A+\sin A\cos B-\sin B\cos A$ $=2\sin A\cos B$	M1
	$\left[\sin(A+B)+\sin(A-B)=\sin 5x+\sin 3x\right]$ $Ax+Bx=5x$ $Ax-Bx=3x$ $\Rightarrow A=4x,\quad B=x$	M1
	$\Rightarrow \sin(4x+x)+\sin(4x-x)=\sin 5x+\sin 3x=2\sin 4x\cos x$	A1 cso [3]
	ALT	
	$\sin(A+B)-\sin(A-B)=\sin A\cos B+\sin B\cos A-(\sin A\cos B-\sin B\cos A)$ $=2\sin B\cos A$	[M1
	$\left[\sin(A+B)+\sin(A-B)=\sin(A+B)-\sin\left[-(A-B)\right]=\sin 5x+\sin 3x\right]$ $\Rightarrow A=x,\quad B=4x$	M1
	$\Rightarrow \sin(x+4x)-\sin(x-4x)=\sin 5x-\sin(-3x)=\sin 5x+\sin 3x=2\sin 4x\cos x$ $\left[-\sin(-3x)=+\sin 3x\right]$	A1cso]
(b)	**Intersections on *x*-axis** $6\sin x\cos x=0\Rightarrow \sin 4x=0,\quad \cos x=0$	M1
	from $\sin 4x=0\Rightarrow 4x=0,\pi,2\pi,3\pi,4\pi\rightarrow x=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{2},2\pi$ **and**	M1
	from $\cos x=0\Rightarrow x=0,\frac{\pi}{2},\frac{3\pi}{2}$	A1
	$\Rightarrow x=0,\frac{\pi}{4},\frac{\pi}{2}$	
	Integration $(3)\int(\sin 5x+\sin 3x)\,dx=(3)\left(-\frac{\cos 5x}{5}-\frac{\cos 3x}{3}\right)$	M1 A1 ft
	Limits $\int_0^{\frac{\pi}{4}}(\sin 5x+\sin 3x)\,dx\pm\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin 5x+\sin 3x)\,dx$	M1
	Substitution of values	

	$3\left\{\left\|\left[\left(-\frac{\cos 5\left(\frac{\pi}{4}\right)}{5}-\frac{\cos 3\left(\frac{\pi}{4}\right)}{3}\right)-\left(-\frac{\cos 5(0)}{5}-\frac{\cos 3(0)}{3}\right)\right]-\left[\left(-\frac{\cos 5\left(\frac{\pi}{2}\right)}{5}-\frac{\cos 3\left(\frac{\pi}{2}\right)}{3}\right)-\left(-\frac{\cos 5\left(\frac{\pi}{4}\right)}{5}-\frac{\cos 3\left(\frac{\pi}{4}\right)}{3}\right)\right]\right\|\right\}$	ddM1
	$=2.73137...+\left\|-1.313170...\right\|=3.8267...\approx 3.86$	A1 cao [8]
		Total 11 marks

Part	Mark	Additional Guidance
(a)	M1	Use of both correct trig identities [either adding or subtracting], allow any two distinct letters or symbols and simplify to reach either $2\sin A\cos B$ or $2\sin B\cos A$ Allow use of $4x$ and x in place of A and B
	M1	Finds or deduces that $A=4x, \quad B=x$ Or if they use the **ALT** scheme then $A=x, \quad B=4x$ This may be implied by the next step or just written down.
	A1cso	For a final correct statement with no errors seen.
(b)	M1	(Placing the equation = 0) and arriving at one of $\sin 4x = 0$ ***or*** $\cos x = 0$
	M1	At least any **one** correct value for x from $\sin 4x = 0$ **and** at least any **one** correct value for x from $\cos x = 0$
	A1	Correctly identifies the required intersections of the curve with the x-axis either explicitly here or used in limits in following work, 0 need not be explicitly stated.
	Note: Award for these values seen without working, and allow working here in degrees.	
	M1	**Integration** An attempt at integration of both terms from their part (a) to an expression of the form $\pm\frac{\cos 5x}{5}\pm\frac{\cos 3x}{3}$ Ignore: • the presence or absence of the multiplier of 3 for this mark • any limits • incorrect notation
	A1	Fully correct integration. Again, ignore the presence/absence of the multiplier of 3. $(3)\left(-\frac{\cos 5x}{5}-\frac{\cos 3x}{3}\right)$
	M1	**Limits** Candidate clearly shows the integration of the curve needs to be done in two parts, where their limits are explicitly shown and come from the limits identified from solving the equation $6\sin x\cos x = 0$.

		As a minimum we need to see two integrals of the function (this can either be written in the original form or the changed form). Their limits must be clearly shown for this mark (following through their solutions) and the upper limit on the first integral must be equal to the lower limit on the second. The candidate does not need to show that the modulus of the second integral must be taken for this mark. Ignore the presence/absence of the multiplier of 3.
	ddM1	Substitution of all limits –dependent on the previous two method marks, allow slips in simplification. The substitution must be correct. If you see any of the decimal answers shown in the working column or any of $\frac{8+4\sqrt{2}}{5}$ (1st integral) or $\frac{4\sqrt{2}}{5}$ (2nd integral) or $\frac{8+8\sqrt{2}}{5}$ (sum of integrals) award this mark. **NOTE: They must consider the modulus of the second integral, and the multiplier of 3 must also be taken into account for this mark.**
	A1cao	For awrt 3.86

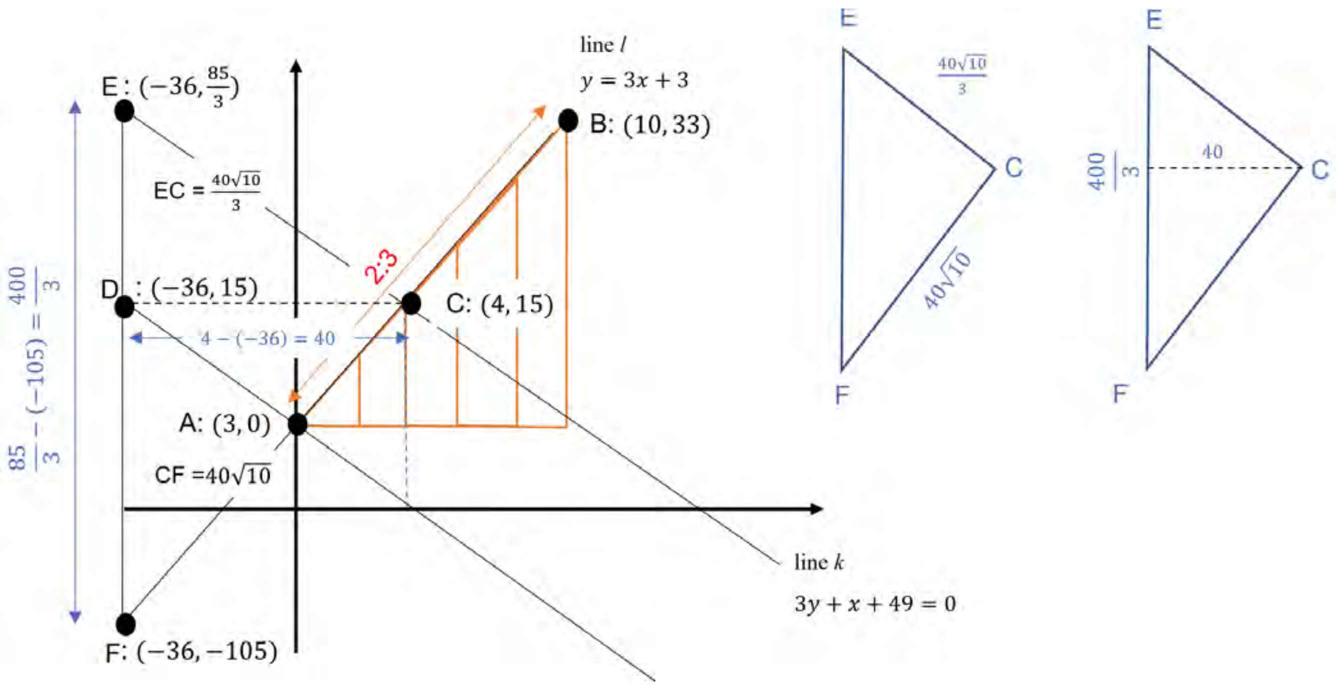
Question number	Scheme	Marks
11 (a)	$b = 33$	B1
	$\left(\left(\frac{3\times 0+2\times 10}{5}, \frac{3\times 3+2\times 33}{5}\right)\right) = (4, 15)$	B1 B1
	ALT	
	Identifies the correct x coordinate of $x = 4$ using any method.	B1
	$y = 3\times '4' + 3 = ... \Rightarrow y = 15$	B1B1
	For the gradient of line k $\quad m = -\frac{1}{"3"}$	M1
	$y - “15” = -\frac{1}{"3"}(x - “4”)$	dM1
	$3y + x - 49 = 0$* cso	A1* cso [6]
(b)	$A = (0, 3)$	B1
	$-\frac{1}{"3"} = \frac{q-"3"}{p(-0)} (\Rightarrow p = -"3"(q-"3"))$ **alt** $y = -\frac{1}{3}x + 3 \Rightarrow q = -\frac{p}{3} + 3$ **OR** $p^2 + (q - "3")^2 = 1440$	M1
	Equation in *q* $10q^2 - 60q + 90 = 1440 \Rightarrow q^2 - 6q - 135 = 0$ oe $10(q-3)^2 = 1440$ or $q^2 - 6q - 135 = 0$	dM1
	Equation in *p* $1440 = p^2 + \left(3 - 3 - \frac{p}{3}\right)^2 \Rightarrow 1440 = \frac{10p^2}{9}$ And attempting to solve by any valid method, reaching a value for p	
	$q - 3 = \pm\sqrt{144}$ or $(q-15)(q+9)$ or $\frac{--6 \pm \sqrt{(-6)^2 - 4.(1).(-135)}}{2\times 1}$	ddM1
	$q = 15, p = -36$	A1 A1 [6]
	ALT	
	$A = (0, 3)$	[B1
	$a\sqrt{(-3)^2 + 1^2}$ $(=12\sqrt{10})$ $\Rightarrow a = (12)$	M1
	$p = 0 - 3\times "12"$ $q = 3 + 1\times "12"$	dM1 dM1
	$p = -36$ and $q = 15$ or D is $(-36, 15)$	A1 A1]

(c)	**Finding the coordinates of points *E* and *F*** (Finding *E* $x = -36$ $3y + x - 49 = 0$) $3y + ("-36") - 49 = 0\ \left(y = \dfrac{85}{3}\right)$ (Finding *F* $x = -36$ $y - 3x = 3$) $y - 3("-36") = 3\ (y = -105)$ **Find the area of triangle *ECF*** **Method 1** $(CE =)\sqrt{("4"-"-36")^2 + \left("\dfrac{85}{3}"-"15"\right)^2} = \left(\dfrac{40\sqrt{10}}{3}\right)$ **Or** $(CF =)\sqrt{("4"-"-36")^2 + ("15"-"-105")^2} = \left(40\sqrt{10}\right)$ $(\text{Area triangle } ECF =)\dfrac{1}{2}\times\dfrac{40\sqrt{10}}{3}\times 40\sqrt{10} = \dfrac{8000}{3}$ oe	M1 M1 M1 M1A1 [5]
	Method 2 Area $= \dfrac{1}{2}\begin{bmatrix} "-36" & "4" & "-36" & "-36" \\ "\dfrac{85}{3}" & "15" & "-105" & "\dfrac{85}{3}" \end{bmatrix}$ oe $\dfrac{1}{2}\left(\begin{matrix}\left["-36"\times"15" + "4"\times"-105" + "-36"\times"\dfrac{85}{3}"\right] \\ -\left["-36"\times"-105" + "-36"\times"15" + +"4"\times"\dfrac{85}{3}"\right]\end{matrix}\right) = \left(\dfrac{10060}{3} - [-1980]\right) = \dfrac{8000}{3}$ oe	M1 M1A1
	Method 3 Finds the lengths: $DC = 4-(-36) = 40$ and $EF = \dfrac{85}{3} - (-105) = \dfrac{400}{3}$ Area $= \dfrac{1}{2}\times '40' \times '\dfrac{400}{3}' = \dfrac{8000}{3}$ oe	M1 M1A1
	Total 17 marks	

Part	Mark	Additional Guidance
(a)	B1	Correct value of b. This may be embedded in a coordinate of B (10, 33)
	B1	For either $x = 4$ or $y = 15$
	B1	For (4, 15) or both $x = 4$ and $y = 15$
	ALT (Without finding the value of b)	
	B1	For $x = 4$ [by any method]
	B1	Substitutes their value of $x = 4$ into the given equation of l $y = 3\times'4'+3 = ...$
	B1	For $y = 15$
	M1	For identification of a gradient from the equation of l and use of $m = -\frac{1}{"3"}$
	dM1	For the correct use of any form of the equation of a line, if using $y = mx + c$, must reach $c = ...$ Note that $c = \frac{49}{3}$ Use of their (4, 15) which **must** have come from an attempt to find the midpoint of AB and their m. **Note:** Dependent upon the previous method mark.
	A1	$3y + x - 49 = 0$ accept terms on one side in any order. Cso **Note:** This equation is given. Award this and the previous mark that has come from work without any errors or omissions.
(b)	B1	Identifies the y intercept of line l (0, 3). This can be implied by later correct work or accept seen in part (a) Look for this coordinate written on a sketch if it is not in the body of the work.
	M1	Equates $-\frac{1}{"3"}$ to a **correct** expression for the gradient in terms of p and q, using **their** values of (0, 3) for the y intercept. **OR** for **correct** use of Pythagoras for the length of AD to form an equation in p and q, using **their** value of (0, 3) for the y intercept.
	dM1	For forming a quadratic equation in either p or q which must come from using their expression for the gradient and the length. **This is dependent on the first M mark in (b)**
	ddM1	For solving their quadratic equation by any acceptable method. **This is dependent on the previous two M marks in (b)**
	A1	For a correct p **or** q.
	A1	For a correct p **and** q.
	ALT	
	B1	Identifies the y intercept of line l (0, 3) This can be implied by later correct work or accept seen in part (a) Look for this coordinate written on a sketch if it is not in the body of the work.
	M1	Uses a correct method, scaling the gradient of $-\frac{1}{3}$ Must arrive at a scaling value (shown as a in the mark scheme), but it may not be the correct value. This mark may be implicit in work that follows.
	dM1	Uses their scaling factor, with the coordinates of A, to find a value for p or q **This is dependent on the first M mark in (b)**
	dM1	Uses their scaling factor, with the coordinates of A, to find a value for p and q **This is dependent on the first M mark in (b)**
	A1 A1	First A1 for p **or** q Second A1 for p **and** q
(c)	M1	Substitutes their value for – 36 into the equation for line k to find the y coordinate of point E
	M1	Substitutes their value for – 36 into the equation for line l to find the y coordinate of point F

	Method 1	
	M1	Uses Pythagoras with **their** values to find the length of *CE* **and** *CF* correctly.
	M1	Correct method to find the area of the triangle, using **their** values.
	A1	For $\frac{8000}{3}$ oe
	Method 2	
	M1	For a correct array using their values.
	M1	Correct calculation shown, using their values, takes the modulus and multiplies by $\frac{1}{2}$
	A1	$\frac{8000}{3}$ oe
	Method 3	
	M1	For finding the lengths *DC* **and** *EF* correctly using **their** values.
	M1	Applies the correct formula for the area of a triangle using their values.
	A1	For $\frac{8000}{3}$ oe

USEFUL SKETCH



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